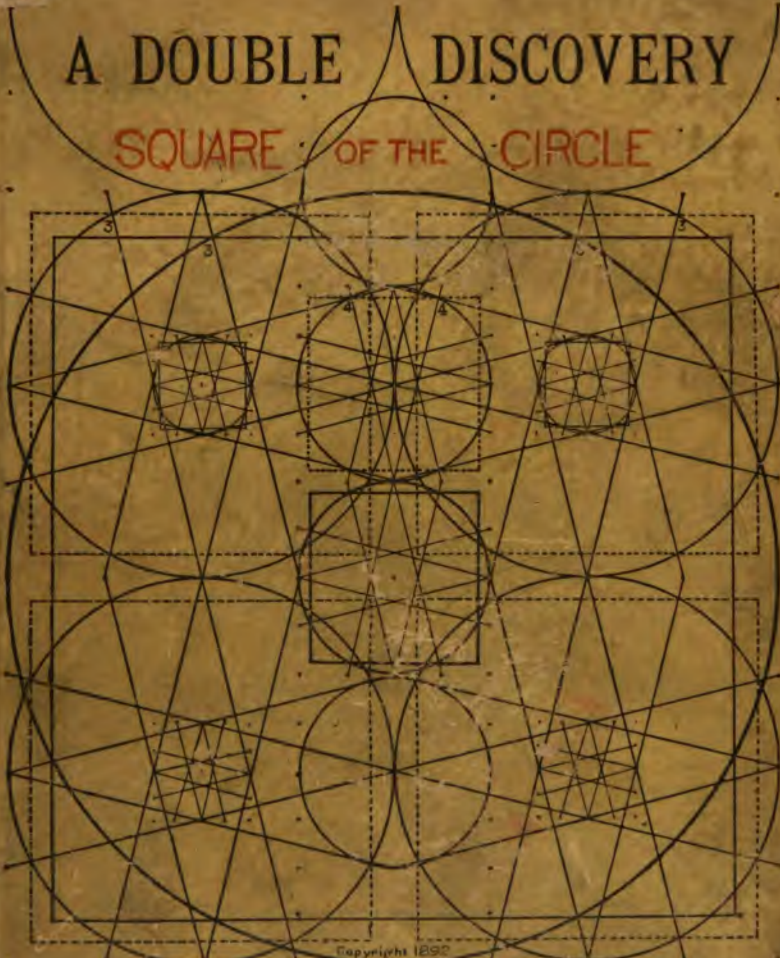
A DOUBLE DISCOVERY

SQUARE OF THE CIRCLE



BY

RUFUS FULLER

THE LONG-SOUGHT-FOR ABSOLUTE FRACTION FOUND, IN PLACE OF INCORRECT APPROXIMATE FRACTIONS, AND IN PLACE OF UNENDING DECIMALS.

BOSTON, MASS.
Printed for the Author
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Col. T. W. Higginson

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PREFACE.

THE diagrams in this work are not found in any ancient or modern works, with the exception of the common circle and simple forms used for illustrating.

The famous problem of Squaring the Circle is solved in Geometry.

In the mathematical part, in place of unending decimals there is shown the absolute fraction now discovered that was searched for by the ancient Greeks, and the ancient problem is solved in Mathematics.

Both parts taken together constitute the Double Discovery.

In the explanations, algebraic signs are not used, but plain terms; and coincidently the solution of this, the most widely known of the intricate problems, was effected during the four hundredth circling quadrangular year, 1892.

The rounded year 1000 being also a year of an ancient historical event in the eastern part of the western half of the world, it is alluded to in connection with the cut of a lofty tower at the end of the diagrams.

Attention is called to the appendix.

RUFUS FULLER.

255 TREMONT ST., BOSTON, MASS.
June, 1893.

LIST OF DIAGRAMS.

SQUARE OF THE CIRCLE.

THE PROBLEM ITSELF, ITS HISTORY AND FINAL SOLUTION IN GEOMETRY AND MATHEMATICS.

DIAGRAM No. 1 represents the circle of the earth as if from a side view, and also the square of the circle. The dotted lines extend from the degrees of the circle of the earth to the circle of time, which in this case would properly be designated by 24 hours on the clock dial. The degrees of the circle of the earth are based upon 6 as a primary number, from which number arise 12 and 24 hours and 360 degrees. A probable reason why 6 was adopted as a primary number in computing the degrees of the circle of the earth was that 60, which is one sixth part of 360, can be divided by ten lesser numbers without a remainder, the divisors being 2, 3, 4, 5, 6, 10, 12, 15, 20, 30.

The squaring of the circle being quadrangular, it is not based upon 6 as a primary number, but

upon 4. The outside and largest square in Diagram No. 1 is divided into four equal parts on each side, designated by dots, and from these dots transverse lines are drawn, beginning at the dot at 360 degrees, which is one half the distance from the left-hand corner of the square. A line is drawn down to the dot which is one fourth the distance from the left-hand corner of the square at the bottom, and where this transverse line crosses the circle a square is drawn crossing at the same point, forming a three-line intersection; and this square has the same area as the circle. This is a newly-discovered fundamental principle in geometry, shown more fully in other diagrams.

Diagram No. 9 has not only nine squares of circles blended together, but also triangles and diamonds, each of which contains the same area as the circle, and containing many intersections of lines at many points.

In Diagram No. 3 this blending is more clearly shown; there is a three-line intersection at the figure 3 of the longest line of one triangle with the shorter line of another triangle and with one line of the diamond. A portion of the longest lines of the triangles in the middle part forms the square that is the square of the circle. The

diamond is half as long as the triangle and twice as wide. The triangle, diamond, square and circle each contain one square inch in area. The dotted lines form two rectangles or oblong squares that cross each other, and each contains double areas, or two square inches each.

Diagram No. 4 is composed of a ring of ten circles overlapping so as to contain three equal spaces at the dotted line as shown in the lower circles, and the line drawn from centre to centre forms one line of the square of the circle across the intervening circles. By trial it is found that this cannot be done with any ring of circles except ten in number.

Diagram No. 5 is of more importance than it might seem to be at first. It was demonstrated by an ancient Grecian, that if a circle is cut at the top, bent down and laid straight, and lines drawn from each end to where the centre of the circle was, as denoted by the dotted lines, an elongated triangle is formed that has the same area as the circle; and carrying the geometrical principle farther than he did, it is also demonstrated that by drawing lines tangent with this same circle, which is the smaller circle, forming a five-pointed star, and then drawing lines from

the points of the star to the centre of the circle, five triangles are formed, each of which contains the same area as the circle; and by drawing a circle around at the points of the star, a portion of which is shown by the curved dotted lines, and then near the smallest circle drawing another circle whose diameter is one-third that of the diameter of the circle at the points of the star, this circle has the same area as the five-square, or pentagon, already formed, whose lines are tangent with the smallest circle.

The uniform whole and half numbers in the dimensions form a very noticeable feature of this diagram. The circle at the points of the star has a diameter of $4\frac{1}{2}$ inches, the five-square is 1 inch on each of the five sides, and the circle containing the same area as the five-square has a diameter of $1\frac{1}{2}$ inches.

Diagram No. 6 is the most noted of all. There are four sizes of squares of circles. The dots at the terminations of the lines of the smallest size are $\frac{1}{8}$ of an inch apart, the dots of the next larger size $\frac{1}{4}$ of an inch apart, of the next larger size $\frac{1}{2}$ inch apart, and of the largest size of all, relatively, 1 inch apart.

Where the figure 3 is seen three lines intersect,

and where the figure 4 is seen four lines intersect. Some of the squares of the circles are made with dotted lines to distinguish them from other lines that are near to them; the square of the largest circle of all is a full line. It takes four in area of any one size to make one of the next larger size in area.

In each size there are equally spaced terminations of lines at the dots on the exteriors, and multiple intersections of lines at single points in the interiors, and all of symmetrical order as contained in the larger whole. And the problem is solved in geometry.

In Diagram No. 2 four lines intersect at single points, some of them at right angles; and it takes sixteen of one size in area to make one of the next larger size in area, and in the same relative proportion there is an increasing of size sixteen-fold in area interspacing on to the circle of the universe.

A square differs from a circle in two important points relative to the equal area of each.

First. The breadth of the square must be less than the diameter of the circle, and the length of the four lines of the square added together must be longer than the length of the line of the circle.

Second. The breadth of the square and the sum of the four lines of the square may both be a whole number. But the diameter of the circle and the length of the circle itself cannot both be a whole number; one or both must be a mixed number. Therefore the principal part of the problem is in the circle itself.

There are two rules for computing the area of a square. The first rule is to multiply two sides of the square, as $8\times 8=64$.

The second rule is to add the length of the four sides together, multiply the sum by the breadth of the square, and divide the product by 4. The result is the same, $8+8+8+8=32$; $\times 8=256$; $\div 4=64$.

The circle not being divided into four parts like a square, in computing the area of a circle the second rule applies. Multiply the length of the circumference of the circle by the diameter, and divide by 4. But here occurs the main feature in the problem, being the relative lengths of the circle and the diameter which must first be found. And now we come to the mathematical part of the problem, which is the other part of the double discovery.

The third book of the elements of Euclid per-

tains in the main to circles, but nothing definite is set forth concerning the squaring of the circle, probably for the reason that he could not effect it.

One of the earlier recorded attempts to solve the problem was by Archimedes, another ancient Grecian philosopher and mathematician, who lived 2200 years ago. He demonstrated by an elaborate process with polygons that the line of the circumference of a circle is three times as long as the diameter and a fraction more than that, which fraction stands between $\frac{10}{70}$ and $\frac{10}{71}$. In this there is no problem. The problem is to find the fraction that stands between $\frac{10}{70}$ and $\frac{10}{71}$.

This fraction has been searched for since that time, and eminent mathematicians have declared that there must be such a thing as a square and circle having the same area; that there must be such a fraction in place of unending decimals; and, although they have not found it, that it may yet be found.

Decimals express decimal values, and a value that exists between decimals may be expressed by a fraction in place of decimals that cannot be reached by an extension of decimal figures: $333\frac{1}{3}$ is a fixed number of definite value that cannot be expressed in decimals.

In making use of decimals, it is known that 3.1415 is too short for the length of the circle, the diameter being 1, and 3.1416 is only an infinitesimal amount too long. In extending other decimals to great lengths beyond the figure 5, the exact length has never been attained by the use of decimals. It is an infinitesimal amount less than 3.1416.

The diameter of a circle must be more than 1 inch that the circle may contain 1 square inch in area.

It is now stated and explained at length that the diameter of a circle being $\frac{167}{148}$ inches, the area of the circle is the same as the area of a square that has 1 inch on the side, being 1 square inch; and the diameter of a circle being 167 inches, the area of the circle is the same as the area of a square that is 148 inches long on the side, being 21904 square inches.

It is for mathematicians to determine for themselves that the above figures ($\frac{167}{148}$ for the diameter of the circle in the one case, the side of the square being 1; and the diameter of the circle being 167 inches in the other case, the side of the square being 148 inches) show not a mere coincidence, but a new discovery in mathematics;

for, as further shown, this discovery was one of the means by which the exact length of the circle was discovered, the diameter being 1.

Computing first for the diameter 167 inches long, as shown below:—

3.1416	
167	
219912	
188496	
31416	
524.6472	
167	
	148
36725304	148
31478832	
5246472	1184
	592
4)87616.0824	148
21904	21904

The diameter of the circle is 167 inches, neither more nor less. The first product obtained, 524.6472, is the assumed length of the circle for that diameter, but too long because 3.1416 is too

long; and by the rule for computing, multiplying that assumed length of the circle by the diameter, 167, and dividing by 4, the same figures are produced in square inches for the area of the circle that are produced for the square that has 148 inches on the side, — 21904 square inches; with the difference that in computing for the area of the circle, the decimals .0824, in the second product, stand in the way, because 3.1416 is an infinitesimal amount too much at the beginning. Reversing the process, leaving out the decimals .0824, dividing 87616 by 167, there is produced $524\frac{108}{167}$ instead of 524.6472; dividing $524\frac{108}{167}$ by 167, in place of 3.1416 there is produced $3\frac{3949}{27889}$, which is the length of the circle, the diameter being 1, and is not an infinitesimal amount too long or too short.

Therefore, the diameter of the circle being 167 inches, computing with 3.1416, which is too long, the area of the circle obtained is 21904.0206.

Computing with $3\frac{10}{70}$ of Archimedes, which is too long, the area obtained is 21912.75571+.

Computing with $3\frac{10}{71}$ of Archimedes, which is too short, the area obtained is 21898.75704+.

Computing with $3\frac{3949}{27889}$, the area is 21904, the same as the square of 148 times 148.

In like manner, the diameter of the circle being $\frac{167}{148}$, computing with 3.1416, which is too long, the area obtained is 1.00000094+. Computing with $3\frac{10}{70}$ of Archimedes, which is too long, the area obtained is 1.0004011+. Computing with $3\frac{10}{71}$ of Archimedes, which is too short, the area obtained is .99976606+. Computing with $3\frac{3949}{27889}$, the area is 1 square inch.

Another proof is given, in which the area is between the very small one of 1 square inch and the very large one of 21904 square inches.

The most ancient record that is known concerning the problem of the square of the circle has come down from the ancient Egyptians, more ancient than that of the ancient Grecians. There has been found in the British Museum in London a book called the Papyrus Rhind; the translator states that it must have been written between 2000 and 1700 B. C. It is stated in this papyrus that a square is equal in area to a circle if the length of the side of the square is 8 and the diameter of the circle is 9.

The reality in this case is, that the area of the square is 64, and the area of the circle is $63\frac{17217}{27889}$; but it is to be noted that these figures 8 and 9 of the Egyptians form the nearest approx-

imation that can be stated in whole numbers, that is, in low numbers not extending up into hundreds. The lowest figures in whole numbers are not the approximate figures 8 and 9 of the Egyptians, but the absolute figures 148 and 167.

Now we can overcome the approximation of the Egyptians and make it absolute. What 167 is to 148, and what $\frac{167}{148}$ is to 1, $9\frac{1}{37}$ is to 8, in the resulting area.

The diameter of a circle being $9\frac{1}{37}$, and computing with 3.1416, which is too long, the area obtained is 64.000060189+. Computing with $3\frac{10}{70}$ of Archimedes, which is too long, the area obtained is 64.0256704+. Computing with $3\frac{10}{71}$ of Archimedes, which is too short, the area obtained is 63.983754+. Computing with $3\frac{3949}{27889}$, the area is 64 square inches, the same as the area of the square of 8 times 8.

A table is given below, containing various fractions of the ancients for the length of the circle besides those of Archimedes, together with the names and dates of those who have produced them, showing their various approximations, when computing with the diameter of $9\frac{1}{37}$, the areas of some being less than 64, and of others more than that; none could reach just 64: —

Archimedes, Grecian, 300 B. C.	$3\frac{10}{71}$	$63\frac{95710}{97199}$
Archimedes, Grecian, 300 B. C.	$3\frac{10}{70}$	$64\frac{246}{9583}$
Chinese, Ancient,	$3\frac{7}{50}$	$63\frac{66228}{68450}$
Ptolemy, Alexandrian (Egypt), 150 A. D.	$3\frac{17}{120}$	$64\frac{233}{164280}$
Hesse, German, 1776 A. D.	$3\frac{14}{99}$	$63\frac{135026}{135531}$
Bhaskara, Hindoo, 1200 A. D.	$3\frac{177}{1250}$	$64\frac{103}{1711250}$
Metius, German, 1597 A. D.	$3\frac{16}{113}$	$63\frac{154684}{154697}$
The Author, American, 1892,	$3\frac{3949}{27889}$	64

Mathematicians have often noted the fact that the approximation of Metius, $3\frac{16}{113}$, is the nearest that has ever been made with a fraction in the history of the world, as will be seen in this case, the result being more than 63 and so near 64 that the first four figures in the numerator and the first four in the denominator are the same, and the last two figures in each are near in value. The last mixed number in the above table gives an area of 64, neither more nor less; the circle is the same as the area of the square of 8 times 8.

Still another result is shown. In reducing $9\frac{1}{37}$ to fractional form we have $\frac{334}{37}$, and in reducing 8 relatively to $9\frac{1}{37}$ we have $\frac{296}{37}$; therefore, being stated in other whole numbers, the side of the

square is 296, and the diameter of the circle is 334; and by taking 296 for a square root, 296 $\times$ 296, we have 87616 for a numerator; and by taking one half of 334, which is 167, for a square root, 167 $\times$ 167, we have 27889 for a denominator, thus, $\frac{87616}{27889}$; and this fraction reduced to a mixed number is $3\frac{3949}{27889}$, which is the length of the circle, the diameter being 1.

It will be observed from the above figures, — the length of the diameter in the larger case, $9\frac{1}{37}$, and the length of the circle in the smaller case, $3\frac{3949}{27889}$, — that the longer diameter and the shorter circle are correlatively derived each from the other, and 64, the larger area, is a quadrangular as well as a rounded number that can be reduced in even numbers that are also rounded numbers down to the unit.

Computing with decimals, the diameter being $9\frac{1}{37}$, the figures given below are very important as another demonstration of the solution of this problem: —

3.1415926, too short, area 63.999904385682+.
3.1415970, too short, area 63.999999074506+.
3.1415971, too long, area 64.000001111687+.
$3\frac{3949}{27889}$, exact length, area 64.

The seven places of decimals in the first row at

the left were produced 300 years ago, and mathematicians have been accustomed to think that there could be no other arrangement of the first seven places of decimals than those; and because an extension of decimals beyond those seven figures in the first row does not produce the exact value nor come to an end, some have said that the problem cannot be solved in mathematics.

It will be seen that the decimals of the second row are of a value longer than those in the first row, and yet too short; and the third row, containing the figure 1 in place of the cipher above it, is too long. And as we formerly found the curious feature of the figures $\frac{167}{148}$ and 1, and 167 and 148, pertaining to straight lines, so in the length of the circle is the other curious feature of the figures 70 and 71 of Archimedes, the value being between the two in the two rows of decimals also, as well as between the fractions of $3\frac{10}{70}$ and $3\frac{10}{71}$. The fact is to be noted also that when used as denominators of fractions, 70 represents a longer value than 71, and when used as decimals, 71 represents a longer value than 70; and the fraction of $3\frac{3949}{27889}$ is the exact length between 70 and 71, either as the denominators of the fractions, or as the last two figures in the above seven decimals.

As $333\frac{1}{3}$ is a fixed number of definite value that cannot be expressed in decimals, the same is true of $3\frac{3949}{27889}$; the fraction $\frac{3949}{27889}$, if reduced to decimals, becomes the middle row of the above seven decimals, and also appears to be unending; but as a fraction it is as absolute as $333\frac{1}{3}$.

Two rules are now given to square the circle. Both rules produce the same result.

First Rule. — Multiply the given diameter of the circle by $3\frac{3949}{27889}$, thereby obtaining the length of the circle for that diameter; then multiply the circle by $\frac{1}{4}$ of the diameter. Or, what is the same, multiply the length of the circle by the diameter and divide by 4. The number obtained is the area of the circle, and the square root obtained from the same number is the length of one side of the square that contains the same area as the circle.

Example. — The diameter of the circle being $9\frac{1}{37}$ inches: $9\frac{1}{37} \times 3\frac{3949}{27889} = 28\frac{60}{167}$; $\times 9\frac{1}{37} = 256$; $\div 4 = 64$. The square root of 64 is 8.

Note. — In the above example the length of the circle is $28\frac{370740}{1031893}$; reduced to lowest terms, dividing by 6179, it is $28\frac{60}{167}$.

Second Rule. — Multiply the given diameter of the circle by 148, divide the product by 167,

and the quotient is the length of one side of the square that contains the same area as the circle.

EXAMPLE. — The diameter of the circle being $9\frac{1}{37}$ inches, $9\frac{1}{37} \times 148 = 1336$; $\div 167 = 8$.

Each rule produces 8 for the square root, and 64 square inches for the area of both the circle and the square.

It is well known that in this nineteenth century of discoveries many things have been accomplished that were formerly supposed to be impossible, and for this reason it would be generally conceded that the solution of this problem being possible, it was desirable that it should have been accomplished before the closing of the present century. Before the year 1492 the problem of the circle of the earth and the nature of its surface area was supposed by the great body of the people of that time to be one that could not be solved. Previous to the year 1892, no astronomer, geographer, navigator, architect, civil engineer, or mechanic had ever been able to compute the circumference of a circle from the diameter or the diameter from the circumference, except approximately. Attempts to do this have been made during so long a period, and by men of so many different nationalities and races in Europe,

Asia, and other parts, that this statement is made in Webster's Unabridged Dictionary, under the word "square:" "The solution of this famous problem [the squaring of the circle] is now generally admitted to be impossible," — a qualified statement, but quite proper when it was made. Since the telephone has come into use, no one would think of making the following assertion, in connection with the word "sound": The solution of the problem of ordinary conversation by two persons in different cities is generally admitted to be impossible!

The problem of squaring the circle is of a triple nature, including two straight lines; diameter of circle, side of square, the circle itself. The fundamental figures of each having been found, they are found once for all. The long-distance telephone as such was searched for but a few years before it was found, but the figures for the triple nature of this mathematical problem have been searched for for hundreds and thousands of years. Any one could compute the relative length of one straight line to another straight line; but the straight line of the diameter of a circle and the straight line of the side of a square being involved in this problem, no one had previously

been able to compute the relative lengths of those two straight lines, from the time of the ancient Egyptians until the year 1892. During that year it was accomplished after many months of persistent labor on the problem, and is now published, together with the newly discovered geometry, for the benefit of the present and future generations.

In Diagram No. 12 the diameters of the smallest circle and the largest circle at the right are whole numbers, and the lengths of the circles are mixed numbers. The same figures that give the length of the line of the smallest circle, $3\frac{3949}{27889}$, also give the surface area in square inches and fraction thereof of the largest circle; the diameter being twice as long, the circle twice as long, and the area four times as large as the area of the smallest circle,—the area, $3\frac{3949}{27889}$, being four times the area of $\frac{21904}{27889}$.

In the lower circle the diameter is not a whole number but a mixed number, and the length of the circle is a whole number. In this size of circle the same figures that give the length of the diameter also give the surface area of the same circle. There is more than 1 square inch in the circle, and yet the length of the circle is 4 inches, the same as the length of the 4 sides of the

square that contains an area of 1 square inch. If this circle is pressed like a hoop into the form of a square it will contain 1 square inch in area, but a less area than it contained as a circle. In like manner, if the 1-inch square is pressed into the form of a circle it will contain more than 1 square inch in area. The principle of this will be seen by pressing the tire of a wheel into the form of a carriage elliptic spring; there is not as much space in it then, although there is the same length of the tire as before, and if pressed close together there is no space left.

If the area of the lower circle is reduced to an improper fraction, the numerator will be the same as the denominator of the smallest circle, and the denominator the same as the numerator of the smallest circle.

The numerator of the area of the smallest circle is 21904. If the length of the circle, $3\frac{3949}{27889}$, is reduced to an improper fraction, the numerator is 87616, and both numerators have the same denominator, 27889. And if the numerator 87616 is divided by the numerator 21904, the quotient is 4, which is the quadrant number.

Diagram No. 10 is a ten-pointed star, and has peculiar dimensions. The triangle that has only

one end extending to the outer circumference, as denoted by the dotted lines, has the same area as the triangle that extends to the outer circumference at both ends, as denoted by the lines formed of dashes. This will be seen by cutting a paper the size of one, and it will fit the other, and both contain the same area as the circle. The longest line of one triangle is through the centre of the circle, and the longest line of the other triangle is at the circumference of the circle. The longest line of still another triangle is within the circle, and it has a smaller area.

Diagram No. 11 contains a triangle, the lines of which are 1 inch in length, and all the lines are 1 inch in length from the four-square and five-square to the eleven-square. The circle is 11 inches in length, but it contains a larger area than the eleven-square. The design is to show that the equilateral triangle contains a less area in proportion to the sum of the lines of which it is formed than any other form that has equal lengths of sides, and the circle, being a single continuous line, contains a larger area in proportion than any other form that can be made.

The possibility of these 1-inch lines being continued, or by gradually shortening the straight

lines that a circle of itself would be formed, is a problem of itself.

It was suggested by a man of extensive learning of the last century that on other planets there may be geometrical principles that are not known on this earth, and it might be added that possibly there are geometrical principles known here that are not known there.

The sage of Concord begins Essay X thus: "The eye is the first circle, the horizon which it forms is the second, and all our lifetime we are reading the copious sense of this first of forms."

The circle pertains to another problem, concerning which the remark is often made that perpetual motion cannot be found. But although it may not be determined to have been found by mechanical appliances on a small scale, it is already found on the larger scale in the revolving worlds of the universe; and the circle—or its general features—is a form of substance and a form of motion that pertains to other worlds than ours, as perceived in astronomy. It is also observed that on our own earth, in the three kingdoms of nature, the circular or curving form predominates in the animal kingdom, appears also in the vegetable kingdom, while in the rocks and other

substances of the mineral kingdom the sharper and angular forms are prominent.

It is recorded that the very near approximate figures 3.1416, which have been so generally used, were produced by Arybahatta, a Hindoo mathematician, about A. D. 500. About A. D. 1597 the figure 5 was substituted for the 6 by Van Cuelen, a resident of Holland, but a native of Germany, who extended the line of decimals to 35 places, and found that if he placed the figure 8 or the figure 9 at the 35th place of the decimals the value was between the two; and he gave it up there.

In the eighteenth century, Mr. Shanks, of Durham, England, carried the decimals to 527 places. Dr. Rutherford reviewed them to 411, and found the computation correct to that point, and Mr. Shanks afterwards carried them to 607 places, with no better final result.

Of the attempts at geometrical solution, one was by rolling a marked cylinder over a plane surface also marked, — an operation similar in principle to that of the wheelwright who rolls a small marked wheel in the inside of a tire to get the right size of the tire.

About two hundred years ago a Frenchman drew a circle and two diameters crossing each

other, and then cut the diameters through the centre of the circle of sufficient length to turn over four corners, forming a square. In this case a large amount of money was involved between him and others who contested the claim of the solution. They were probably those who knew that it was not mathematically demonstrated, nor geometrically, since there was no definite place for the folding, and since, also, the amount taken up by the folding was not defined. The case was taken into court, and from the court was appealed to the king, who set the case aside. The time had not come, in that century, for courts or kings to be able to judge of the merits of the case in hand.

About the same time an Italian lawyer went to London with the claim that he had solved the problem. He said that the side of a given square being 5 inches, the diagonal line of the square is 7 inches. In that size the error is so small that it might deceive many in the first drawing; but when his claim was examined, it was shown that he had made an error amounting to the difference between 49 and 50.

Whatever the size of the square, the diagonal line is never commensurate with the side in whole

numbers. But there is a rectangle or oblong square in which the diagonal line is commensurate in whole numbers. If an oblong square is formed 4 inches long and 3 inches wide, a diagonal line between the corners is 5 inches, neither more nor less. Then, by erasing one line of the length of the oblong square and one line of the width, a triangle of unequal sides is left; and by drawing a square from the outside of each of the three lines of the triangle there is formed in outline the noted 47th proposition of Euclid. Then, by filling the three squares so formed with one-inch squares, the 9 squares on one side and 16 on another, making 25, are balanced by the 25 squares on the other side of the triangle, being 50 squares in all, not 49; and in a certain sense, in this case, 5 equals 7.

On taking the 47th proposition of Euclid where he left it, and where it has remained for some 2300 years, and extending it as further shown, many wonderful results are obtained. Diagram No. 13 shows one of these results, which embraces the three principles of uniformity, diversity, and symmetry combined, and illustrates the familiar line of the poet Keats: —

"A thing of beauty is a joy forever."

In Euclid's figure, the central part is the unequal triangle. If the largest square is taken for the central nucleus, the other parts are only on one side; but by extending those on the four sides various attractive forms can be produced that are not found in windows of ancient or modern architectural designs.

Diagram No. 14 exhibits another result, derived from an extension of Euclid's 47th proposition; the drawing being neither a square, nor a triangle, nor a diamond, but a combination of all. The outline, composed of straight lines, contains an area of 173 small squares. The line of the circle is outside the corner of one square and inside the corner of another square, not touching either. We have now come to the principle of corners and another form of inside and outside measurements, different from that of a circle and its connection with a common square. In this case, instead of being the even and rounded number of squares, 64, the area is the odd and pointed number, 173.

Other interesting combinations are derived from Euclid's 47th proposition, and also from Archimedes' triangle extended, that are not embodied in this work.

Of other Americans who have given their attention to the problem of squaring the circle, there was one who published a book in 1851, in which it was disclosed that the side of a square being 5153, the sum of the length of the four sides is 20612; and the diameter of a circle being 6561, the length of the circle is 20612.01821+. As he was under the necessity of admitting that there is in this case, also, the unending line of decimals more than 20612 in the length of the circle, he endeavored to overcome it with the specious plea that the circumference of a circle is more than the circle itself, as the tire of a wheel is larger than the wheel. Whatever name is applied to the line of the circle, the mathematical principle is a line that is necessarily conceived to be a line without thickness, or an imaginary line. His result was like that of the ancients, being another one of the very near approximations in figures applied in this case, but not absolute.

Another more recently merely transferred the unending line of decimals from the circle to the diameter, and published that in a periodical as a solution of the problem.

The practical utility of absolute figures may be conceived of in engineering and mechanical work.

In boring a tunnel for a double railroad track through the Hoosac Mountain in Massachusetts, which took twenty years' time to complete, the work proceeded from the east and from the west. The meeting within the mountain varied somewhat from the correct line; but with the absolute diameter and circle measurement and trigonometry combined, such engineering work, and measuring the height of mountains, the coast survey, and calculations in astronomy, may be made more accurate.

Diagram No. 8 is a new design, representing the electric light. These diffusive cross-lines furnish another newly discovered principle in geometry, derived from the whole number of points being an odd number. One of the squares of the circle in this work is a five-square in a five-pointed star.

The very large star used on the signal flags by the ocean steamship companies at sea is a five-pointed star; being of this form it can be distinguished at a long distance at sea as a star.

It is taught in an Etymological Reader used in the schools that in 1777, during the Revolutionary War, a committee of the Congress called upon Mrs. Ross, who kept a shop on Arch Street,

Philadelphia, and asked her if she could make a United States flag, according to a plan they would produce, — the design being thirteen red and white stripes, alternate with thirteen six-pointed stars. She consented, and suggested that the stars should be made with five points instead of six. At her suggestion this was agreed to.

It would seem to be fortunate that this change was made, for it is readily seen that a six-pointed star, commonly made by placing two equilateral triangles across each other, does not produce a star-like appearance, but is more like a mariner's sextant.

In the State House in Boston is a marble slab, which is a fac-simile of one that is in a very ancient church, a few miles from Stratford-on-Avon, in England. On this slab are the name of Lawrence Washington — great-great-grandfather of George Washington — and the family coat of arms, consisting of alternate horizontal stripes; above the stripes are three five-pointed stars, with blank circles in the centres.

These three five-pointed stars, ancestrally noted, were made thirteen stars, in Philadelphia, on the Star-Spangled Banner, that at first represented the thirteen original States, and are now increased

to forty-four stars, representing the present number of States. While the form of the five-pointed star serves a good purpose as such on the sea and on land, the most beautiful star is a seven-pointed star. Diagram No. 7 is a newly designed seven-pointed star. The circle is tangent with some of the lesser points of the star, and at a distance it has more of the appearance of the twinkling stars that are seen when looking towards the circle of the universe.

"He telleth the number of the stars, he calleth them all by their names, . . . his understanding is infinite." [Psalm cxlvii. 4, 5.]

The stars signify knowledges which are innumerable to man. The Apocalypse is the Greek name for the closing book of Scripture, which is full of numbers and measurements that even the learned do not affect to understand. The following extract is taken from the "Apocalypse Explained," a work that has been published for more than a century. "Spiritual things are not numbered and measured, but still they fall into numbers and measures as they descend out of the spiritual world or heaven where angels are, into the natural world or earth, where men are, and in like manner when they descend out of the spirit-

ual sense of the word into the natural sense. Hence the literal sense of the Holy Word in that respect is such as we find it."

On learning the inner meaning of Scripture numbers and other arcana never made known before, contained in the voluminous works by the same author which are found in the public libraries, the inception was formed of entering upon this present work, pertaining to measurements and problems in the natural world. But pertaining to measurements in another world a passage is here adduced found in the beautiful Nineteenth Psalm, which psalm has more than a spiritual import, being one of the celestial psalms:—

"In them hath he set a tabernacle for the sun. . . . His going forth is from the end of heaven and his circuit unto the ends of it, and there is nothing hid from the heat thereof."

This work begins with the circuit and lines of the earth, and ends with the circuit and heat of the Spiritual Sun, and end unto ends of Heaven.

NOTE.

The diameter of a circle being 1, the length of a circle is $\frac{87616}{27889}$. These figures are referred to again, being a convenient form in computing. And used as whole numbers, the diameter of the circle being 27889, the length is 87616, and the area of the circle is the same as the area of a square that is 24716 on the side, being 610880656.

APPENDIX.

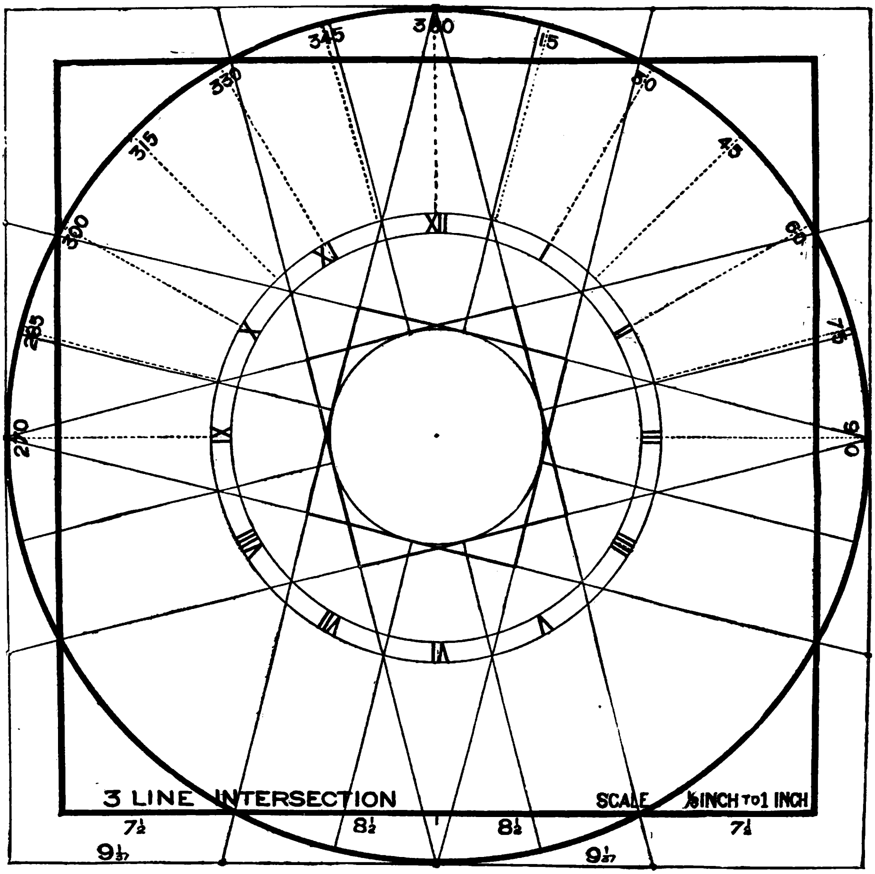
This book, being useful for reference, is bound in a convenient form for the pocket and for mailing.

It is a very suitable gift-book for persons of all ages.

A convenient wrapper for mailing, that will not unloose nor injure the book, is made by cutting a stout piece of paper the length of the book and twice as wide; fold it like the double cover of the book; place the front edge of the book to the back fold of the wrapper, and tie like a package in place of sealing.

The printed regulations of the Postal Department allow the sender to write his own name and address, after the word "From," on the outside of the wrapper, without extra postage.

Postage required for the weight 4 cents.



360
345
15
330
30
315
45
300
60
285
75
270
90
XII
XI
X
II
IX
III
VIII
IIII
VII
V
VI
3 LINE INTERSECTION
SCALE ⅛ INCH TO 1 INCH

DIAGRAM No. 1.

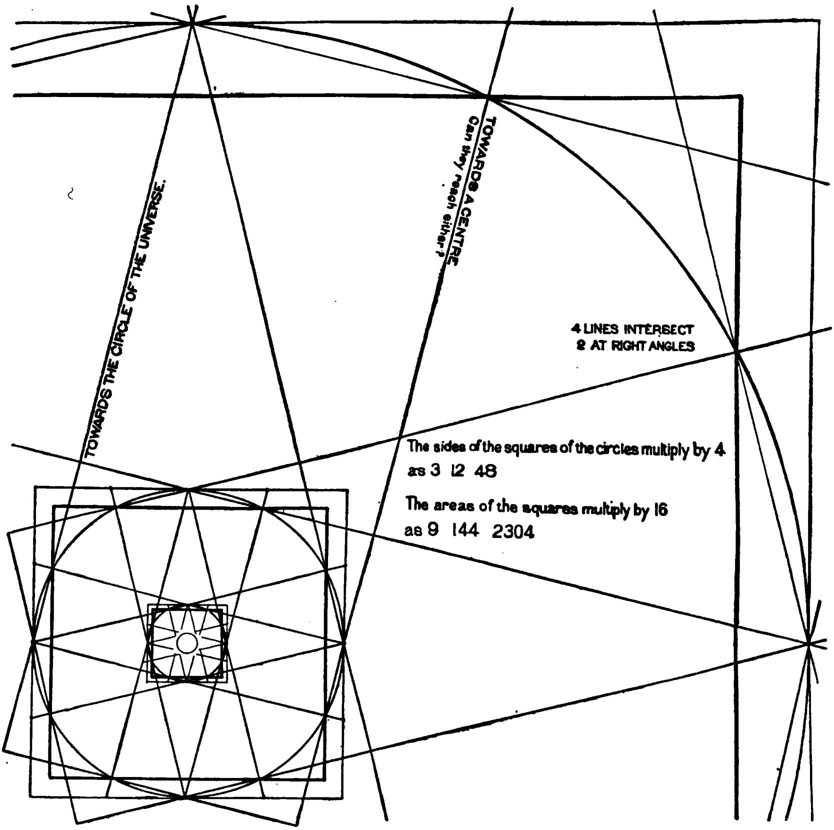

TOWARDS THE CIRCLE OF THE UNIVERSE.
TOWARDS A CENTRE
Can they reach either?
4 LINES INTERSECT
2 AT RIGHT ANGLES
The sides of the squares of the circles multiply by 4
as 3 12 48
The areas of the squares multiply by 16
as 9 144 2304

DIAGRAM No. 2.

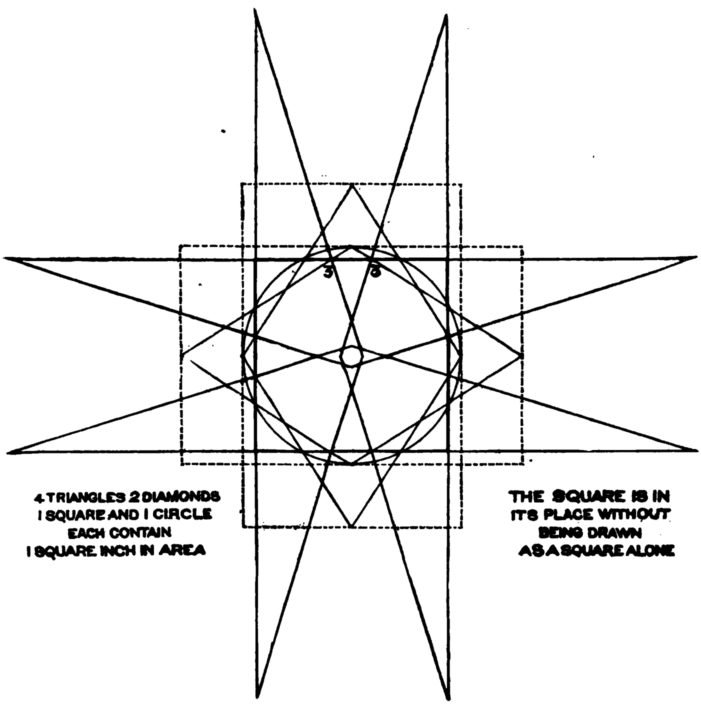
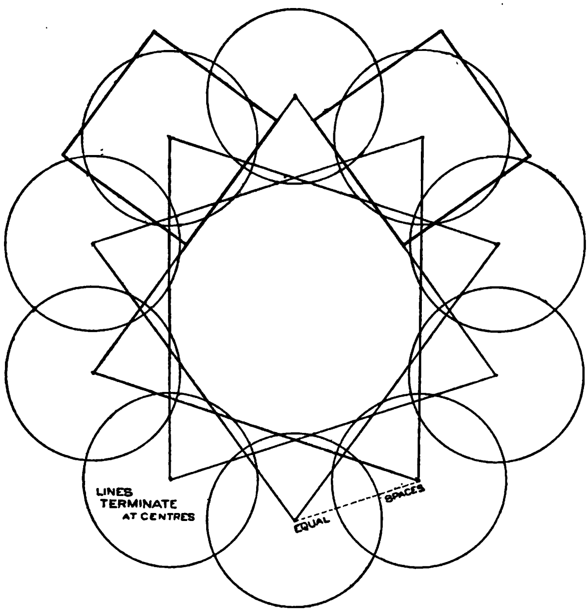

4 TRIANGLES 2 DIAMONDS
I SQUARE AND I CIRCLE
EACH CONTAIN
I SQUARE INCH IN AREA
THE SQUARE IS IN
ITS PLACE WITHOUT
BEING DRAWN
AS A SQUARE ALONE

DIAGRAM No. 3.



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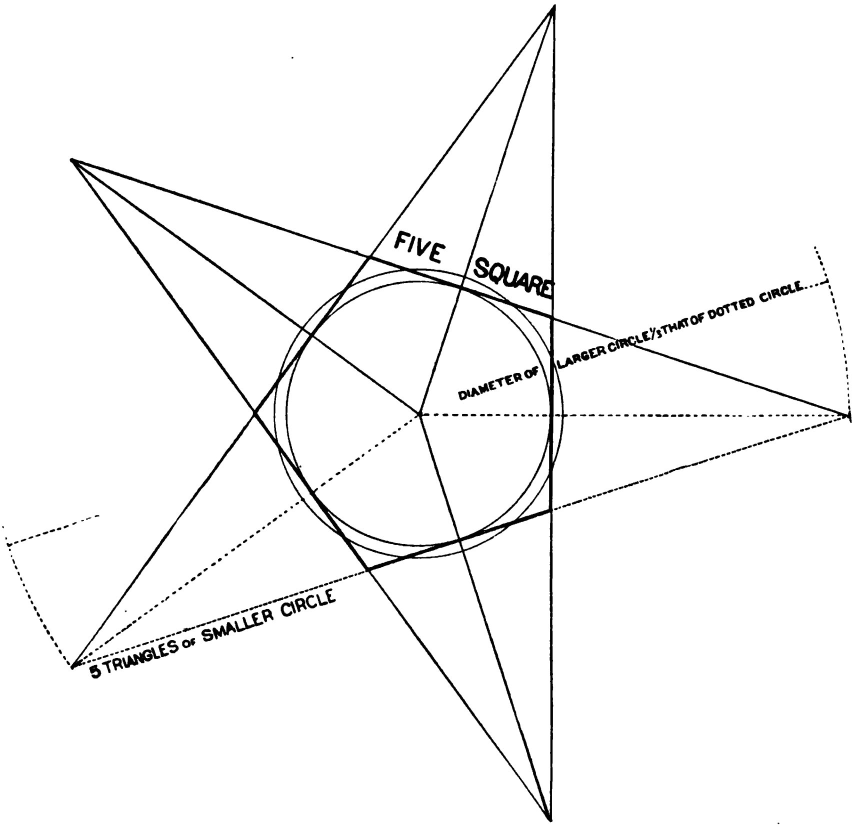
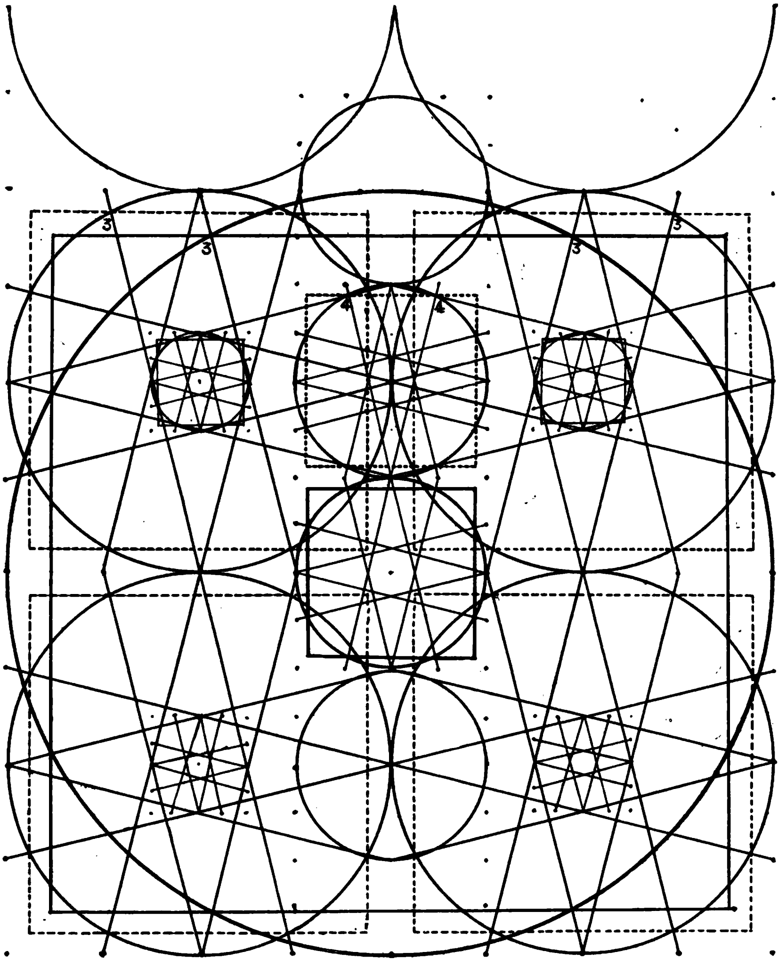

FIVE SQUARE
DIAMETER OF LARGER CIRCLE 1/3 THAT OF DOTTED CIRCLE
5 TRIANGLES OF SMALLER CIRCLE

DIAGRAM No. 5.



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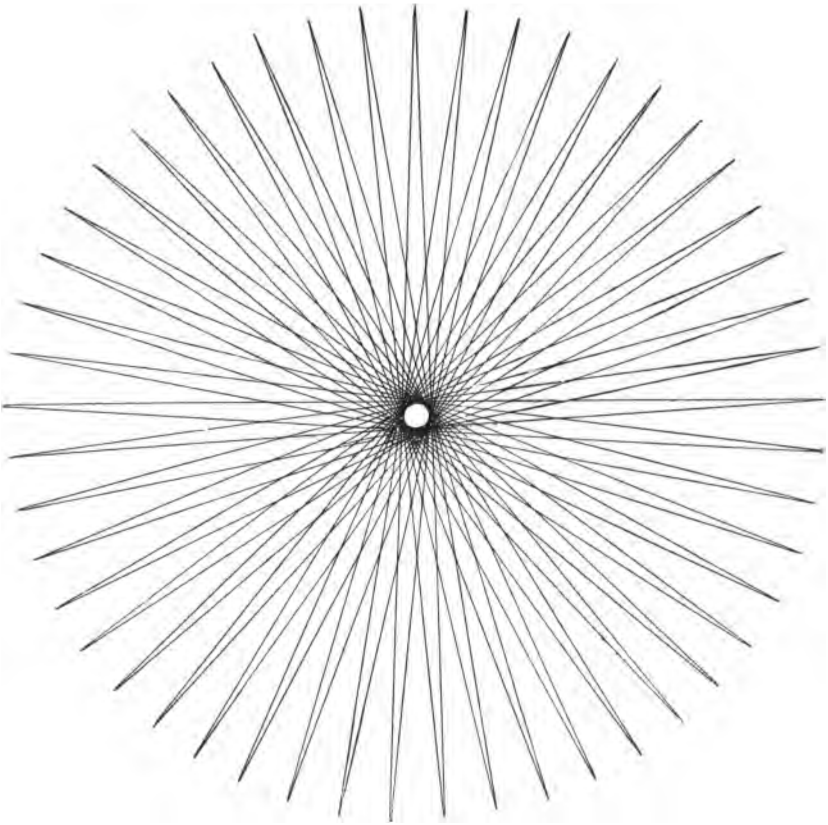

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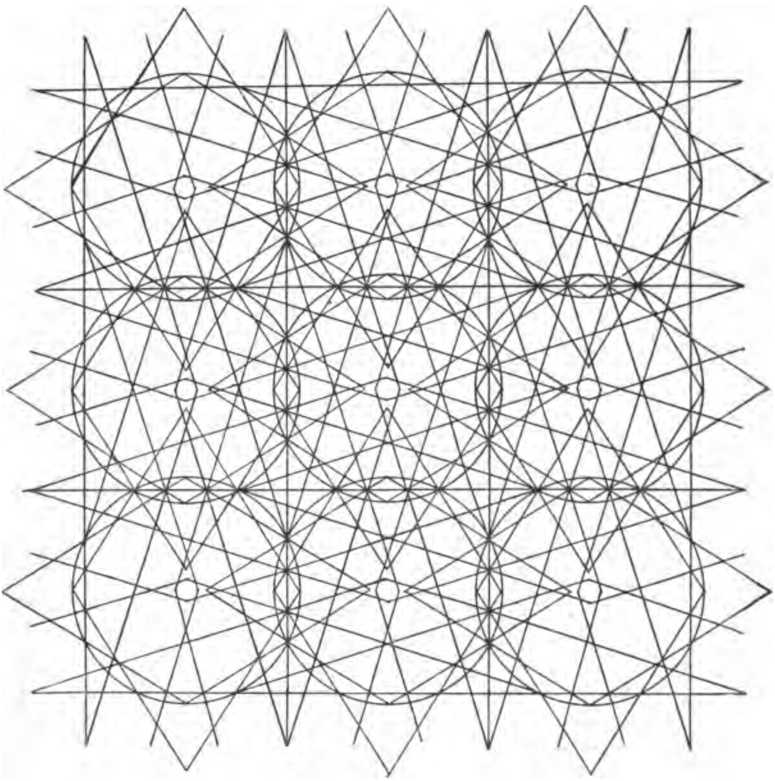

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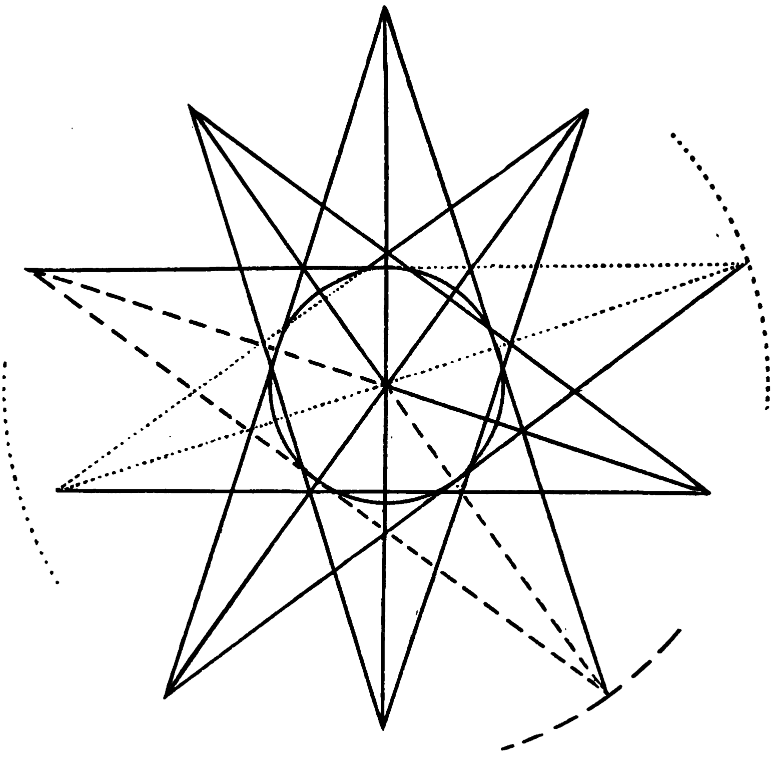

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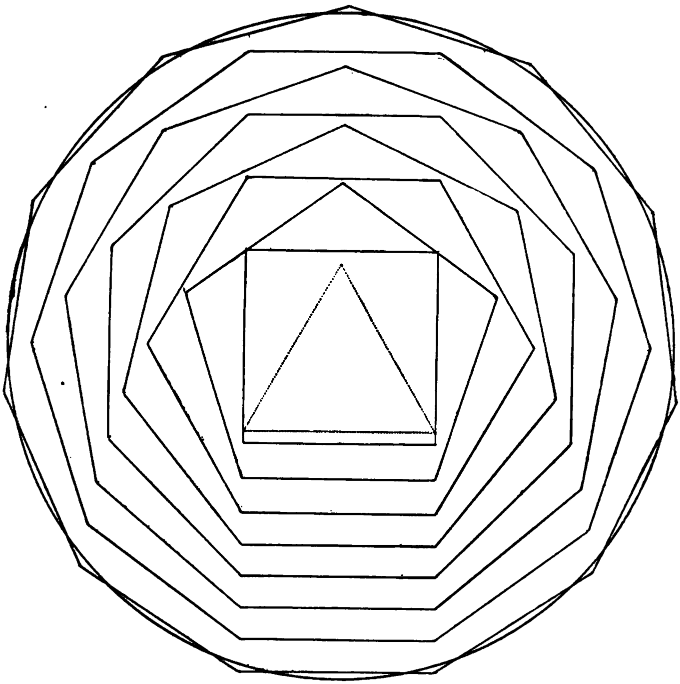

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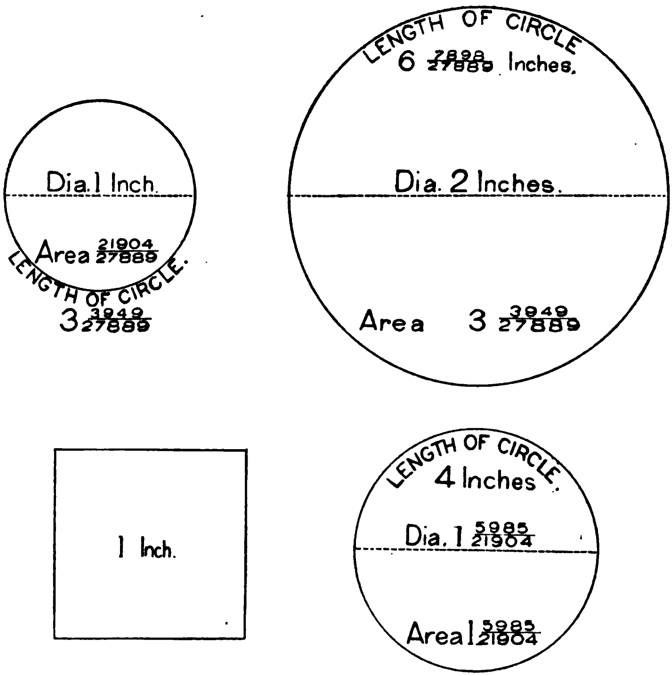

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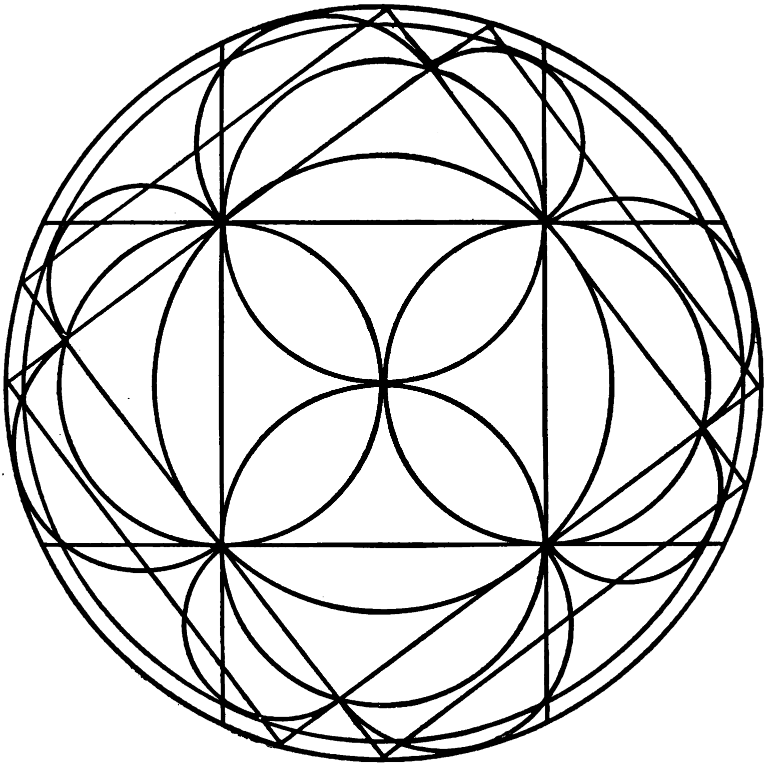

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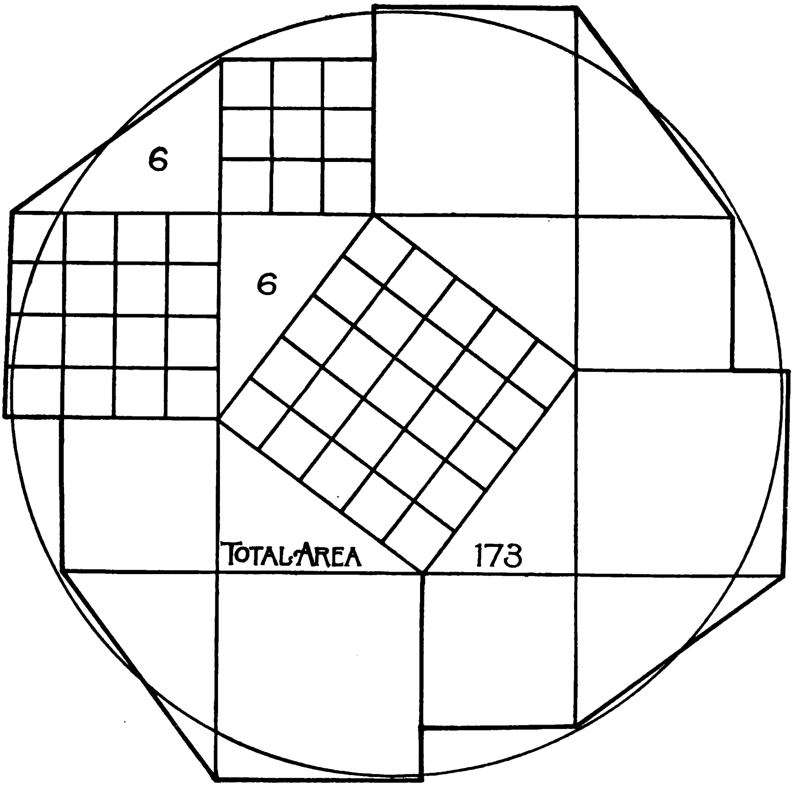

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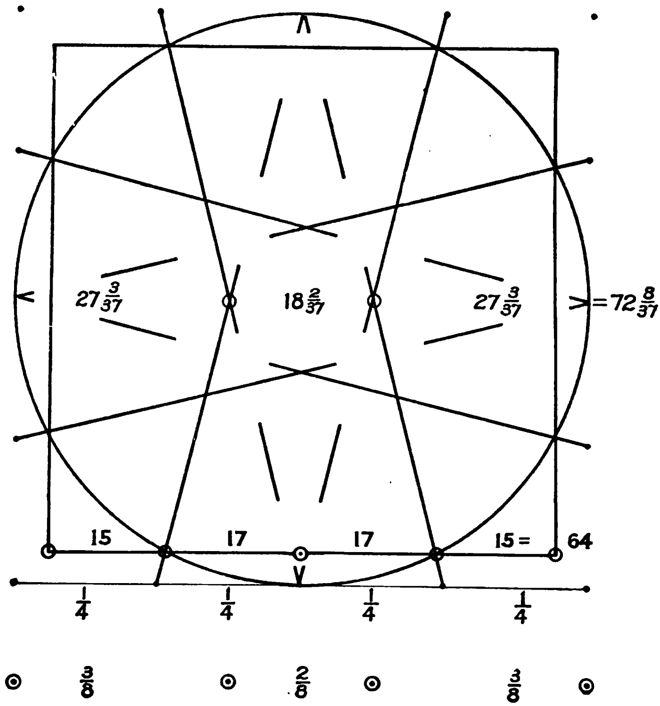

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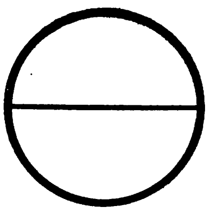

DIAMETER 1.

CIRCLE $3\frac{3949}{27889}$.

DIAGRAM No. 16.

NORSE TOWER

Erected in 1889 on the banks of the Charles River, ten miles from Boston, Massachusetts, U. S. A., by Prof. E. N. HORSFORD (since deceased), of Harvard College, in commemoration of LEIF ERIKSON, the principal navigator, and other Norsemen from Iceland, who made a settlement in these parts in A. D. 1000, the ancient name being Norumbega.

That the settlement was long continued is shown from copious records preserved in Iceland, and from many local evidences, such as a constructed water-way, now dry, twenty-three hundred feet long, a heavy bank wall a thousand feet long, and another wall connecting an island in the river with the mainland, and similar works in adjacent towns.

This part of the present work also ends with recent adjudged discoveries, showing the location of the Caucasian race in the New World in A. D. 1000.